

Realizing Topological Transition in a Non-Hermitian Quantum Walk with Circuit QED

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We extend the non-Hermitian one-dimension quantum walk model [PRL 102, 065703 (2009)] by taking dephasing effect into account. We prove that the feature of topological transition does not change even when dephasing between the sites within units is present. The potential experimental observation of our theoretical results in the circuit QED system consisting of superconducting qubit coupled to a superconducting resonator mode is discussed and numerically simulated. The results clearly show a topological transition in quantum walk, and display the robustness of such a system to the decay and dephasing of qubits. We also discuss how to extend this model to higher dimension in the circuit QED system.

PACS numbers:

I. INTRODUCTION

As a quantum analog of the well-known classical random walk, quantum random walk serves as a fascinating framework for various quantum information processes, such as basic search [1], universal quantum computation [2], quantum measurement [3] etc. Apart from its numerous applications, quantum random walk itself also displays new traits different from its classical counterpart, such as the fast spreading of the wave function compared to a classical random walk [4], which was used for explaining the high efficiency photosynthetic energy transfer assisted by environment [5–8]. Ref. [9] considered an one-dimensional (1D) quantum random walk on a bipartite lattice, where a topological transition was found and experimental implementation in quantum dots or cavity QED (CQED) was briefly discussed. In it, the environment effect was included by using a non-Hermitian

Hamiltonian, as done in [10]. Later, the same model was extended to multi-dimensional systems [11].

As in Fig. 1, the quantum walk could be realized on an 1D bipartite indexed lattice where decay sites (blue) and non-decay sites (white) appear in turn. The strengths of hopping between sites are characterized by v (within the same unit) and v' (between neighboring units), and due to that, a random “walker” starting from a non-decay site of unit m_0 may end up decaying from the system from another unit m with probability P_m . Given the relative strengths of v and v' , the average displacement of a “walker” ($\sum_m m \cdot P_m - m_0$) before it decays is quantized as an integer (0 for $v' < v$ and -1 for $v' > v$) no matter what the decay co-efficient $\gamma > 0$ is. As shown in Ref. [12], such system displayed Parity-Time Symmetry [13–15]. These different displacements correspond to unbroken and broken Parity-Time Symmetry regimes and consist of a topological transition. In this work we will show that such transition is robust even in the presence of qubit dephase. Note that this idea can be experimentally realized in a CQED setting, where a qubit is coupled to one resonator mode [16, 17]. Specifically, the two states of the qubit ($|e\rangle, |g\rangle$) represent two sites of the same unit, where the higher-energy qubit state ($|e\rangle$) has decay rate γ . The cavity photon number state $|n\rangle$ represents the n -th unit. Here we suppose that the cavity decay κ is much less than qubit decay γ and is thus negligible. Such a system could be an atom in a cavity coupled to one cavity mode or similarly, as CQED systems where a superconducting Josephson junction which acts as a qubit, is coupled to a lumped LC oscillator in an superconducting electric circuit. Recently, circuit QED system attracts a lot of attention, as it could be easily scaled up and controlled [18]. We will stick to the circuit QED system setup from now on.

On the other hand, there were experiments realizing bipartite non-Hermitian quantum random walk in optical waveguide [19] with satisfying results, which in essence is a classical simulation of a quantum effect. However, the

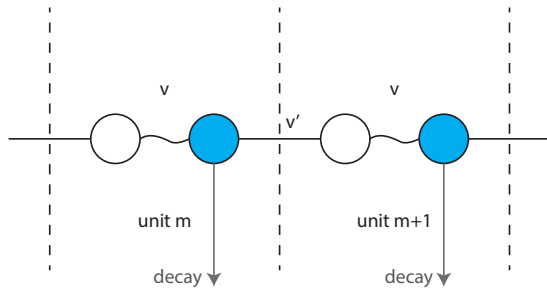


FIG. 1: (color online) Non-Hermitian quantum walk model. A particle can hop between the nearest sites with strength v or v' depending on whether that hop crosses unit boundaries. When the particle is on the blue sites, it would decay with rate γ .

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non-Hermitian quantum walk in circuit QED is a fully quantum experiment. Another merit of circuit QED system is that extension to higher dimension quantum walk is relatively easy, as a high-dimensional quantum walk can be implemented by coupling the qubit to more than one resonator mode. We will show that novel features of a quantum walk emerges with the growth of dimensionality.

In this paper, we first extend the analytical theory in [9] to accommodate the existence of dephase, and show that the qubit dephase, one major source of decoherence in normal circuit QED systems, won't affect the topological structure experimentally observed. Numerical calculations are carried out for different parameters of the system, and different factors affecting the results are considered and discussed, including the preparation of the system, the decay rate of the qubit, the detuning of the system and the effect of dephasing. We will show that such quantum walk realization is feasible in circuit QED setting, and good topological transition can be observed with modest requirement on the system parameters afore-mentioned.

II. THE CIRCUIT QED MODEL

We consider a system where a two-level qubit is coupled to a resonator field, with external microwave driving. Such a system can be expressed in standard circuit QED Hamiltonian

$$H_{\text{org}} = g(\sigma^+ \tilde{a} + \sigma^- \tilde{a}^\dagger) + \Omega/2(\sigma^+ + \sigma^-) + \frac{1}{2}\Delta\epsilon\sigma^z, \quad (1)$$

where σ^z, σ^\pm are Pauli operators for the qubit, g characterizes the coupling strength, and $\tilde{a}(\tilde{a}^\dagger)$ are lowering (rising) operators on the resonator mode in a rotating frame, $\Delta\epsilon$ is (real) detuning of the system, consists of the energy differences between the two states of the qubit minus the minimum energy gap for the resonator, Ω is the strength of external drive on the atom.

Furthermore, we consider decay of the excited state of the qubit, together with qubit dephase. We use γ to characterize qubit decay strength, and d as the dephase factor of the qubit. The effects of such two terms on the system can be added as Lindblad operators as

$$\begin{aligned} \dot{\rho}(t) = & -i[H_{\text{org}}, \rho(t)] + d[\sigma^z \rho(t) \sigma^z - \rho(t)] \\ & + \gamma[\sigma^- \rho(t) \sigma^+ + \sigma^+ \sigma^- \rho(t) + \sigma^- \sigma^+ \rho(t)], \end{aligned} \quad (2)$$

In usual experimental setup [20], the strength of cavity decay is at the magnitude of ~ 1 kHz or less, which is significantly smaller than other parameters in the system, such as the coupling strength g which could reach ~ 100 MHz and qubit decay γ is in the order of MHz. Therefore, we ignore the cavity decay effects. Besides, the external drive Ω is easily tunable to fulfill the topological transition conditions.

Since we are only interested in the state of the system when the qubit first decays, we followed the quantum

jump approach in [21] and put the qubit decay term into the Hamiltonian, which yields an effective non-Hermitian Hamiltonian

$$H = g(\sigma^+ \tilde{a} + \sigma^- \tilde{a}^\dagger) + \Omega/2(\sigma^+ + \sigma^-) + \frac{1}{2}\Delta\epsilon\sigma^z - \frac{1}{2}\frac{i\gamma}{2}|e\rangle\langle e|. \quad (3)$$

We put subscript “org” to distinguish the original Hamiltonian H_{org} from this Hamiltonian that we will work with from now on.

Ideally, if the system is initialized at photon number state $|N\rangle$ (which means the “walker” starts at exact site N), and stays close to N during the whole evolution process, then we can use the approximation

$$\tilde{a} \approx \sum \sqrt{N}|n-1\rangle\langle n|, \tilde{a}^\dagger \approx \sum \sqrt{N}|n\rangle\langle n-1| \quad (4)$$

to simplify the operators. The conditions on which this approximation holds will be discussed later. Then, assuming $\Delta\epsilon \approx 0$, we have

$$v = \Omega/2, v' = g\sqrt{N}. \quad (5)$$

Thus, when $v' < v$ ($\frac{\Omega/2}{g} > \sqrt{N}$) the average photon number upon measurement should be N (no change), and when $v' > v$ ($\frac{\Omega/2}{g} < \sqrt{N}$) the average photon number upon measurement should be $N-1$ (change of -1).

Due to the existence of decay term $-\frac{i\gamma}{2}|e\rangle\langle e|$ the Hamiltonian is not Hermitian. However, it satisfies the condition of Parity-Time symmetry, that is, after undergoing parity ($\hat{x} \rightarrow -\hat{x}, \hat{p} \rightarrow -\hat{p}$) and time reversal ($\hat{p} \rightarrow -\hat{p}, i \rightarrow -i$) transformation, the new Hamiltonian H' is only a constant away from the old Hamiltonian ($H' = H + cI$). And recent work [12] has proven that when we have $\frac{\Omega/2}{g} > \sqrt{N}$ the eigenvalues of the Hamiltonian are totally real, while $\frac{\Omega/2}{g} < \sqrt{N}$ there's a pair of complex energy eigenvalues, which correspond to unbroken and broken PT symmetries respectively.

III. ANALYTICAL THEORY

The analytical theory for this topological transition has been explained in detail before [9]. Here, we will improve their analytical theory to include the qubit dephase. The Lindblad master equation of the system is [22]

$$\dot{\rho}(t) = -i[H, \rho(t)] + d[\sigma^z \rho(t) \sigma^z - \rho(t)], \quad (6)$$

where d characterizes dephase rate for qubit.

If the system is pure, one could use ψ_n^A to denote the amplitude at $|g\rangle \otimes |n\rangle$ and ψ_n^B at $|e\rangle \otimes |n\rangle$. In the presence of dephase the density matrix of the system is mixed. We can similarly use density matrix as $\rho_{n_1, n_2}^{AA}(0) = \rho_{n_1, n_2}^{BA}(0) = \rho_{n_1, n_2}^{AB}(0) = 0$, $\rho_{n_1, n_2}^{BB}(0) = \delta_{n_1, 0} \delta_{n_2, 0}$, where ρ_{n_1, n_2}^{AA} is the matrix entry corresponding to $|g\rangle\langle g| \otimes |n_1\rangle\langle n_2|$ and so can be interpreted for AB ,

BA and BB . We have

$$\langle \Delta n \rangle = \sum_n n \cdot \left(\int_0^\infty \gamma \rho_{n,n}^{BB}(t) dt \right). \quad (7)$$

Then we translate this to momentum space by having

$$\rho_{n,n'}^{C,C'} = \frac{1}{(2\pi)^2} \oint dk \oint dk' e^{ikn} e^{-ik'n'} \rho_{k,k'}^{C,C'} \quad (8)$$

where $C, C' \in \{A, B\}$, the Hamiltonian becomes separable as for a 2×2 subspace of $|g/n\rangle \otimes |k\rangle$ we have

$$H_k = \begin{pmatrix} \epsilon_A & v_k \\ v_k^* & \epsilon_B - i\gamma/2 \end{pmatrix} \quad (9)$$

with $v_k = \frac{\Omega}{2} + g\sqrt{N}e^{-ik}$ and $\epsilon_B - \epsilon_A = \Delta\epsilon$.

Using integration by part, we have that

$$n\rho_{n,n}^{BB} = \frac{i}{(2\pi)^2} \oint dk \oint dk' e^{i(k-k')n} \partial_1 \rho_{k,k'}^{BB}. \quad (10)$$

Summing over n and integrate over time, we have

$$\langle \Delta n \rangle = i\gamma \int_0^\infty dt \oint \frac{dk}{2\pi} \partial_1 \rho_{k,k}^{BB}, \quad (11)$$

where we use $\partial_1 \rho_{k,k}^{BB}$ as a shorthand of $\frac{\partial}{\partial k'} \rho_{k',k}^{BB}|_{k'=k}$.

Now, one can define $p_k(t) := \rho_{k,k}^{AA} + \rho_{k,k}^{BB}$ as the probability that the subsystem of momentum k hasn't decay till time t , with $\partial_t p_k = -\gamma \rho_{k,k}^{BB}$. Also, one can use polar decomposition as $\rho_{k,k'}^{BB}(t) = u_{k,k'}(t) \cdot e^{i\theta_{k,k'}(t)}$. With that, one can write

$$\begin{aligned} \langle \Delta n \rangle = \frac{i\gamma}{2\pi} \int_0^\infty dt \oint dk \left(e^{i\theta_{k,k}(t)} \partial_1 u_{k,k}(t) \right. \\ \left. + u_{k,k}(t) \cdot i \cdot e^{i\theta_{k,k}(t)} \cdot \partial_1 \theta_{k,k}(t) \right). \end{aligned} \quad (12)$$

Considering the fact that $\theta_{k,k}(t) = 0$ as the diagonal terms of a density matrix are real, and $\partial_1 u_{k,k}(t) = \partial_2 u_{k,k}(t)$, one can find that the first term $e^{i\theta_{k,k}(t)} \partial_1 u_{k,k}(t)$ induces an integration of a closed contour, and is thus zero. We can reach

$$\langle \Delta n \rangle = \frac{1}{2\pi} \int_0^\infty dt \oint \frac{dk}{2\pi} (\partial_t p_k(t) \cdot \partial_1 \theta_{k,k}(t)). \quad (13)$$

Defining

$$I_0 = \oint \frac{dk}{2\pi} (p_k \partial_1 \theta_{k,k}(t)|_0^\infty), \quad (14)$$

we have through integration by part

$$\langle \Delta n \rangle = I_0 - \int_0^\infty \oint \frac{dk}{2\pi} p_k \partial_t \partial_1 \theta_{k,k}(t). \quad (15)$$

Given the way we conduct Fourier Transfer, we have $\rho_{k,k'} = \rho_{-k',-k}$, which means $\theta_{k,k'} = \theta_{-k',-k}$. Thus we have

$$\begin{aligned} \oint \frac{dk}{2\pi} p_k \partial_t \partial_1 \theta_{k,k}(t) &= - \oint \frac{dk}{2\pi} p_k \partial_t \partial_2 \theta_{-k,-k}(t) \\ &= \oint \frac{dk}{2\pi} p_k \partial_t \partial_2 \theta_{k,k}(t). \end{aligned} \quad (16)$$

Here we also use the fact that p_k is even function in k . Given the fact that $\partial_1 \theta_{k,k}(t) + \partial_2 \theta_{k,k}(t) = 0$, that integration yields zero, and we have $\langle \Delta n \rangle = I_0$.

Given that the system eventually decays completely, $p_k(t \rightarrow \infty) = 0$, we have

$$\langle \Delta n \rangle = - \oint \frac{dk}{2\pi} \frac{\partial \theta_{k',k}(0)}{\partial k'} \Big|_{k'=k}. \quad (17)$$

Since $\rho(0)$ is pure and diagonal in the basis of σ^z , for $\rho(\epsilon)$ consider up to the first order of $\epsilon \rightarrow 0^+$, one could find that the dephase term $d(\sigma^z \rho(t) \sigma^z - \rho(t))$ doesn't affect $\rho(\epsilon)$ to the first order. Thus, by treating $\rho(\epsilon)$ as a pure state (which is similar to the case in Ref. [9]), we have

$$\langle \Delta n \rangle = - \oint \frac{dk}{2\pi} \frac{\partial \arg(-iv_k^*)}{\partial k}. \quad (18)$$

The topological structure is that, if $\Omega/2 > g\sqrt{N}$, then the integration of $-iv_k^*$ doesn't contain the axis origin and $\langle \Delta n \rangle = 0$; if $\Omega/2 < g\sqrt{N}$, then the integration of $-iv_k^*$ is an anti-clockwise contour of a circle centered at $-i\Omega/2$ and with radius $g\sqrt{N}$, which contains the axis origin, thus $\langle \Delta n \rangle = -1$.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We use a simple numerical integration technique to do the simulation. A $\text{MAXN} = 320$ [23] (which is the total dimension of the Hilbert space under consideration) dimensional complex vector V (matrix, if dephase is in consideration) is used to store the state of the system. In the absence of dephase, a matrix $U = e^{-i \cdot H \cdot t}$ is computed and $U \cdot V$ simply yields the new state vector V . In the presence of qubit dephase we use Lindblad master equation (6). Integration is carried out along the way until the amplitude of V converges [24]. The time interval t shrinks by half each time and Richardson Extrapolation is carried out until the final result converges.

A first glimpse of the results are provided in Fig. 2 with $N = 100$ (which is large, as proposed in [9] to meet the assumption of (4)). Good topological transition is observed if the system is initialized in a Fock state (black).

Since large number Fock states are usually hard to prepare, we considered an alternative: coherent state (Poisson distribution), which the authors of [9] have also chosen by a classical driving. However, the simulation (red

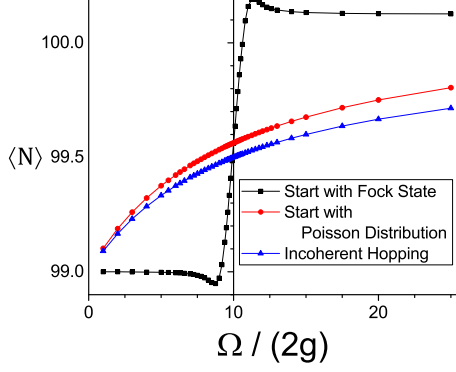


FIG. 2: (color online) Results of a basic simulation for $N=100$ with different starting distributions (Poisson / Fock) of the resonator, and comparison with classical incoherent hopping. The independent variable $\Omega/2g$ captures the relative strengths of inter and intra unit hopping in the quantum random walk, and $\langle N \rangle$ is the average resonator energy upon qubit decay. The other parameters are $g = 1$ (set as a benchmark), $\gamma = 4$, $\Delta\epsilon = 10^{-4}$ while Ω varies.

line in Fig. 2) turns out to be quite similar in the case of non-coherent hopping (classical walker) despite large initial N . This shows that, contrary to the Ref. [9], the Fock states are indeed necessary for testing topological transition.

Considering the fact that high energy Fock states are hard to come by, we relax the condition of $N \gg 1$ and examine some cases with small Fock numbers. An extreme case of $N = 1$ is displayed in Fig. 3(a) which still preserves the basic traits of a topological transition despite the small initial Fock number. Further investigation

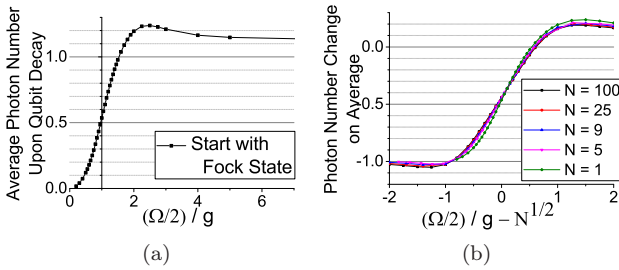


FIG. 3: (color online) Results of simulation for small initial N 's and comparison. The parameters in both figures are (same as Fig. 2) $g = 1$, $\gamma = 4$, $\Delta\epsilon = 10^{-4}$ while Ω varies. (a) Results of simulation with $N = 1$. (b) Displaying change in “decay energy” with respect to $\Omega/2g - \sqrt{N}$

into the problem yields Fig. 3(b), which shows that the curve for different N 's overlap.

A. Different Decay Factors and Energy Level Differences

In this section we consider the effect of qubit decay factors γ and detuning $\Delta\epsilon$ on this experiment.

In theory [9], qubit decay factor γ only affects the (expected) evolution time, not the topological transition, so the expected result should be the same for different γ 's. We run a simulation for different initial N 's with identical parameters like g and $\Delta\epsilon$ for different decay factors γ 's, with result in Fig. 4. We observe that the qubit de-

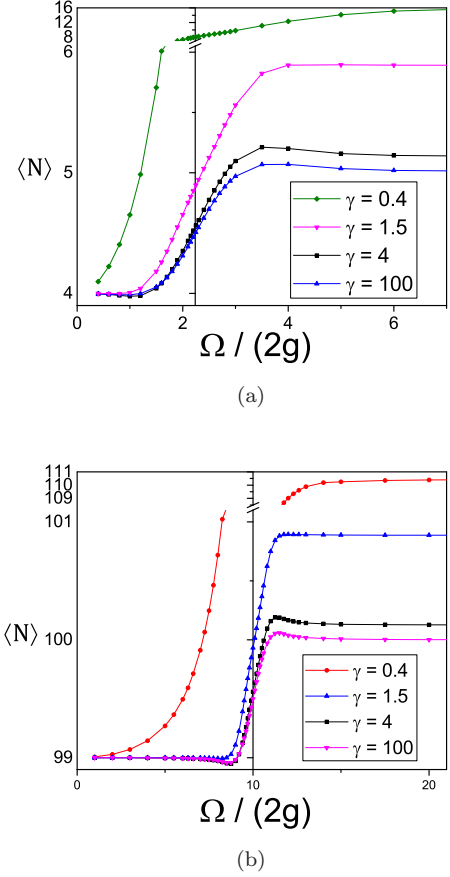


FIG. 4: (color online) Results of simulation with different qubit decay factor γ 's under (a) initial $N = 5$ and (b) initial $N = 100$. The independent variable $\Omega/2g$ captures the relative strengths of inter and intra unit hopping in the quantum random walk, and $\langle N \rangle$ is the average resonator energy upon qubit decay. The other parameters are $g = 1$, $\Delta\epsilon = 10^{-4}$ while Ω varies. One can observe that if qubit decay factor γ is small, the curve significantly deviates from theoretical deduction, while for large γ 's, the curves converge.

cay factor γ cannot be too small (at least $\gamma > g$ should be satisfied), otherwise the photon number upon measurement would be too large. This is necessary for both small and large N 's. We found that $\gamma = 4$ (4 times the coupling strength) is roughly where qubit decay factor starts to negatively affect the observed topological transition,

we will stick to this value from now on.

The discrepancy between analytical theory and numerical results lies in the assumption of equation (4), which is only valid if the system stays close to photon number N throughout the whole evolution. If decay factor γ is small, the quantum walker can walk far away from the initial site, overturning that assumption. Specifically, since we have

$$\tilde{a} = \sum_n \sqrt{n} |n-1\rangle \langle n|, \tilde{a}^\dagger = \sum_n \sqrt{n} |n\rangle \langle n-1|, \quad (19)$$

the strengths of inter-site hoppings grow stronger as the site number (or energy level in our simulation) increases, which explains why for small γ 's the curves deviate significantly to higher energy levels.

Luckily, big γ 's does not present an experimental difficulty in reality, as big qubit decays are usually easy to generate.

And Fig. 5 shows the effect of detuning on this experiment. Ideally, we want $\Delta\epsilon = 0$, which is an exact match

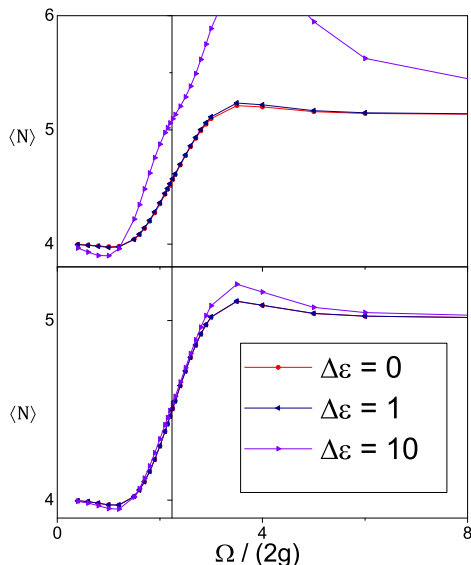


FIG. 5: (color online) Results of Simulation with different System Detuning $\Delta\epsilon$'s under initial $N = 5$, with (upper) under qubit decay $\gamma = 4$ and (lower) $\gamma = 20$. Notice that the difference between $\Delta\epsilon = 1$ and $\Delta\epsilon = 0$ in (lower) is so tiny that one can barely distinguish them in the plot. The independent variable $\Omega/2g$ captures the relative strengths of inter and intra unit hopping in the quantum random walk, and $\langle N \rangle$ is the average resonator energy upon qubit decay. The other parameter is $g = 1$ while Ω varies.

to the quantum random walk scenario. Simulation shows that as long as the detuning of the system is kept small (Fig. 5 shows that $\Delta\epsilon < g, \Omega$ suffices), it won't have a tangible effect on the result of this experiment.

It's worthwhile to notice that in theory, neither $\Delta\epsilon$ nor γ should affect the experimental results. Yet in Fig. 5, one can observe that for a fixed $\Delta\epsilon = 10$, increasing qubit decay γ yields better topological transition. We believe

approximation (4) plays a major role here, just like in Fig. 4, as small qubit decay γ yields wilder amplitude span to overturn approximation (4).

B. Dephasing Effects

Dephasing characterizes one major effect of noise on circuit QED systems, making a quantum system less "quantum" but more "classical". Here, we consider dephase of the qubit due to external field fluctuation, which is the major source of dephase in a circuit QED system, and can be written in Lindblad master equation (6), where d characterizes the strength of such dephase. Such dephase keeps the diagonal elements of ρ , but in addition to other standard evolution the non-diagonal elements of it shrinks exponentially by $e^{-d \cdot t}$. Here, we run a numerical simulation where qubit decay rate γ and other parameters like $g, \Delta\epsilon$ are kept identical for different qubit dephase factor d 's.

The results are in Fig. 6, which shows that the topo-

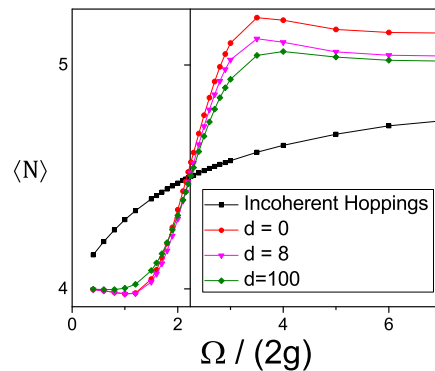


FIG. 6: (color online) Results of Simulation with different Dephase Factor d 's For the Qubit under initial $N = 5$. The independent variable $\Omega/2g$ captures the relative strengths of inter and intra unit hopping in the quantum random walk, and $\langle N \rangle$ is the average resonator energy upon qubit decay. The parameters are (same as Fig. 2) $g = 1, \gamma = 4, \Delta\epsilon = 10^{-4}$ while Ω varies.

logical transition grows less distinct as the qubit dephase factor d grows. Yet even when the dephase of the qubit is very large ($d = 100$) compared with the other elements of the system $\Omega, \gamma, g < 10$, good topological transition can still be observed, as compared to the classical incoherent hopping.

This numerical results agree with analytical theory we put forward earlier in this paper, that this topological structure is protected from the effect of dephase. No matter how much dephase we have in the system, as long as such dephase comes from external field fluctuation, and can be treated as Markovian in the timescale of other operations in the system, it won't affect experimental results.

V. 2-DIMENSIONAL QUANTUM WALK, AND ITS REALIZATION IN A CIRCUIT QED SYSTEM

As an extension to what's previous discussed, we have also investigated 2-dimensional quantum walk. In this case each unit still consists of a decaying site and a non-decaying site, however, a “walker” could make inter-unit hopping in two dimensions, with their strengths characterized by v' and v'' each. The strength of hopping between two sites of the same unit is still v . Fig. 7 provides a sketch of the model.

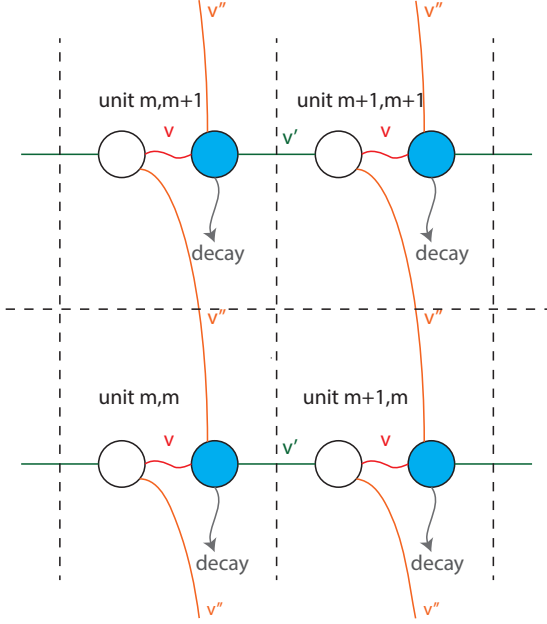


FIG. 7: (color online) Theory setup of two dimensional quantum walk. The system is similar to the one in Fig. 1, except with one more dimension. Three colors (red, orange and green) are used to distinguish different kinds of hopping (red for intra-site, orange and green for inter-site on two different dimensions). The dashed lines are used to indicate the boundaries of each site.

The benefits of a circuit QED system is that it's relatively easy to accommodate this change by adding a second resonator mode coupled to the qubit. Like before, if one uses $|e/g\rangle \otimes |n_1\rangle \otimes |n_2\rangle$ to identify the state of the system, then the Hamiltonian of the system is

$$H = g_1 (\sigma^+ \tilde{a}_1 + \sigma^- \tilde{a}_1^\dagger) + g_2 (\sigma^+ \tilde{a}_2 + \sigma^- \tilde{a}_2^\dagger) + \Omega/2(\sigma^+ + \sigma^-) + \frac{1}{2}\Delta\epsilon\sigma^z - \frac{1}{2}\frac{i\gamma}{2}|e\rangle\langle e|, \quad (20)$$

where σ 's are still Pauli operators on the qubit, and $\tilde{a}_{1/2}$'s are lowering / rising operators on the first or second resonator field in a rotating frame, $\Delta\epsilon$ is (real) detuning of the system, and γ characterizes the decay strength of the excited qubit state.

A. Analytical Theory for Higher Dimensional Random Walk

The theory for higher dimensional case without dephase has been explained in [11], which is a simple extension of the one-dimensional case, and so is the following section considering dephase.

Suppose the dimension is d . As in the one-dimensional case, one can still use $\rho_{\mathbf{n},\mathbf{n}'}^{CC'}$ to denote the state of the system, where \mathbf{n} and \mathbf{n}' are d -dimensional vectors and $C, C' \in \{A, B\}$. The same Fourier Translation into momentum space yields that for subspace \mathbf{k} ,

$$H_{\mathbf{k}} = \begin{pmatrix} \epsilon_A & \mathbf{A}_{\mathbf{k}} \\ \mathbf{A}_{\mathbf{k}}^* & \epsilon_B - i\hbar\gamma/2 \end{pmatrix} \quad (21)$$

with $\mathbf{A}_{\mathbf{k}} = \frac{\Omega}{2} + \sum_{\alpha=1}^d g^{(\alpha)} \sqrt{N} e^{-ik_{\alpha}}$. Defining $p_{\mathbf{k}}(t)$ in the same way as

$$p_{\mathbf{k}} = \rho_{\mathbf{k},\mathbf{k}}^{AA} + \rho_{\mathbf{k},\mathbf{k}}^{BB} \quad (22)$$

and with $\partial_t p_{\mathbf{k}} = -\gamma \psi_{\mathbf{k},\mathbf{k}}^{BB}(t)$ and integrate by part, one can reach

$$\langle \Delta n_{\alpha} \rangle = \oint \frac{d^{d-1}k}{(2\pi)^{d-1}} \left\{ i\gamma \int_0^\infty dt \oint \frac{dk_{\alpha}}{2\pi} \partial_{k_{\alpha},1} \rho_{\mathbf{k},\mathbf{k}}^{BB} \right\}, \quad (23)$$

while the shorthand $\partial_{k_{\alpha},1}$ means taking partial derivative only on the α -th component of \mathbf{k} , and only on the first \mathbf{k} in the density matrix. It's not hard to see that the expression inside the brace is exactly as equation (11) and thus is either 0 or 1, no matter what dephasing factors we have.

Now, if one come back to the definition of $\mathbf{A}_{\mathbf{k}} = \frac{\Omega}{2} + \sum_{\alpha=1}^d g^{(\alpha)} \sqrt{N} e^{-ik_{\alpha}}$, one way to understand equation (23) is to first fix $d-1$ angles $k_{\beta \neq \alpha}$, then see whether the integration of k_{α} from 0 to 2π would case the angle of $\mathbf{A}_{\mathbf{k}}$ to also shift 2π , and integration over those $d-1$ angles. During the integration of the other $d-1$ angles, we may observe new topological structures, like the middle area of Fig. VB, with some old traits still remaining, like the two sides of Fig. VB.

For the 2-dimensional case that is relatively easy to simulate, assuming $g_1 > g_2$, the results are

$$\langle \Delta n_1 \rangle = \begin{cases} -1 & v' > v + v'' \\ -1 + \frac{\theta_1}{\pi} & |v - v'| \leq v'' \\ 0 & v' < v - v'' \end{cases}, \quad \langle \Delta n_2 \rangle = \begin{cases} 0 & v' > v + v'' \\ -\frac{\theta_2}{\pi} & |v - v'| \leq v'' \\ 0 & v' < v - v'' \end{cases}, \quad (24)$$

with $\cos \theta_1 = \frac{N(g_1^2 - g_2^2) - \Omega^2/4}{\Omega\sqrt{N}g_2}$, $\cos \theta_2 = \frac{N(g_1^2 - g_2^2) + \Omega^2/4}{\Omega\sqrt{N}g_2}$, $v = \Omega/2$, $v' = \sqrt{N}g_1$, $v'' = \sqrt{N}g_2$ in this system. One can see a detailed theory deduction in Appendix A.

B. Numerical Results

We run numerical simulations for the 2-dimensional case in a similar manner as the 1-d case, except that the

tensor product of two vectors is stored. We truncated photon number at $\text{MAXN} = 20$ with hindsight knowledge from the 1-dimensional simulations. From Figure VB one can see that the results of the simulation preserve the basic properties of a 2-d transition. It also shows that in the 2-dimensional case, a more stringent requirement is put onto the system as a much bigger decay factor γ is necessary to preserve the topological nature of the system compared to the one-dimensional case.

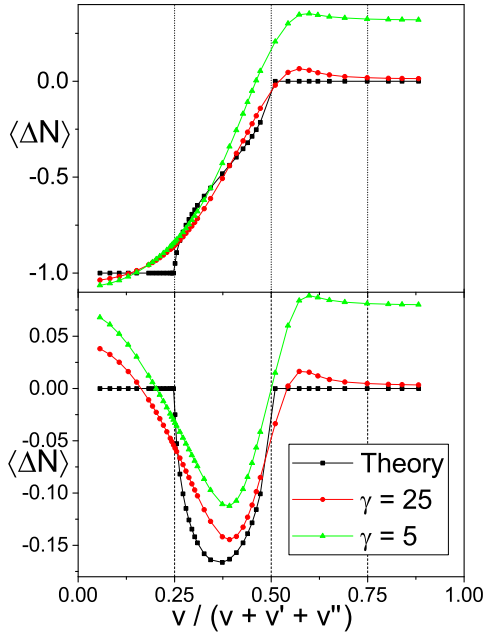


FIG. 8: (color online) 2D Hoppings: Results of simulation with different qubit decay factor γ 's under initial $N = 5$, and comparison with the theoretical (ideal) case for a 2-dimensional random walk in (upper) the first resonator mode, and (lower) the second resonator mode. The coupling strengths are set to $g_1 = 2, g_2 = 1$ to satisfy constrain $v'/v'' = 2$. The independent variable is calculated by $v/(v + v' + v'') := \Omega/(\Omega + 2\sqrt{N}(g_1 + g_2))$ to fit in the form of equation (24). The other parameter is $\Delta\epsilon = 10^{-4}$ while Ω varies. From the fact that a similar qubit decay ($\gamma = 5$) as 1d hoppings leads to large deviation to the ideal topological structure one can observe that a larger qubit decay is necessary for 2d experiments.

We have also numerically simulated the effect of qubit dephase on the final result. Using a similar method with section IV.B, we simulated equation (6) exactly and displayed the results in Fig. VB, which shows that different qubit dephase factor d 's doesn't affect our result.

Due to the fact that analytical theory for different dimensions follow similar integration over contours, we expect that in higher dimensional case, the topological effect would still be protected from qubit dephase.

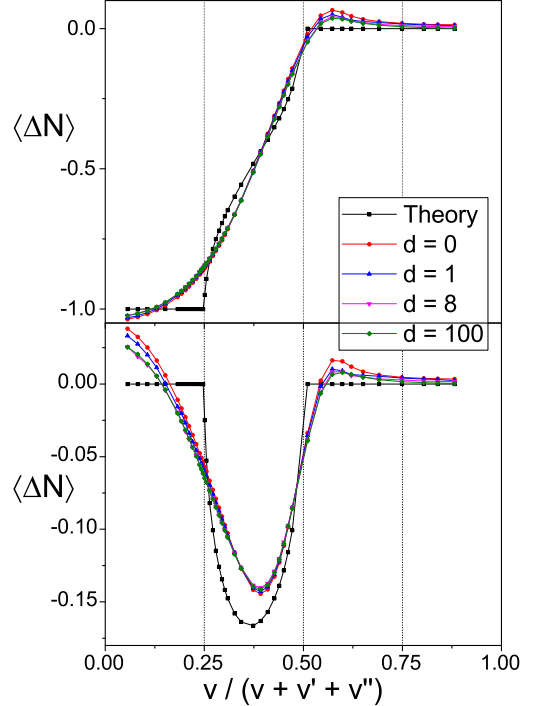


FIG. 9: (color online) 2D Hoppings: Results of simulation with different dephase factor d 's for the qubit under initial $N = 5$, and comparison with the theoretical (ideal) case for a 2-dimensional random walk in (upper) the first resonator mode, and (lower) the second resonator mode. The coupling strengths are set to $g_1 = 2, g_2 = 1$ to satisfy constrain $v'/v'' = 2$. The independent variable is calculated by $v/(v + v' + v'') := \Omega/(\Omega + 2\sqrt{N}(g_1 + g_2))$ to fit in the form of equation (24). $\langle \Delta N \rangle := \langle N \rangle - N$ is the average resonator energy change upon qubit decay. The other parameters are $\gamma = 25, \Delta\epsilon = 10^{-4}$ while Ω varies. From the fact that the curves for different dephase factor d 's almost overlap one can see that qubit dephase factor doesn't affect the simulated result.

VI. CONCLUSION

As a key element of this experimental realization, a measurement of the photon number in the cavity needs to be carried out the instance that the qubit decays. One way to implement such scheme is to prepare a low-decay qubit that's coupled to another resonator mode that has much decay. By continuous measurement of that other resonator mode [25], which provides much of the qubit decay, one can immediately sense the decay of the qubit, and thus carried out a energy measurement in the main resonator mode to retrieve photon number.

In summary, we considered the quantum walk on an one-dimensional bipartite indexed lattice, where each unit has one decay site and one non-decay site. We have proved the topological transition in this non-Hermitian system is not affected by both the decay and the dephasing between decay and non-decay sites within units. We

proposed a circuit QED implementation where a qubit is coupled to one resonator mode. We have shown that quantum random walk can be realized without too much modification to the standard JC Hamiltonian, and that topological transition could be observed to some extent even with small photon number N . We found that there was no topological transition if the systems starts in a coherent state, which is contrary to Ref. [9]. Our numerical results show that the topological structure doesn't

depend on qubit dephase d , and as long as detuning $\Delta\epsilon$ of the system is kept smaller than the coupling strength g , and qubit decay is kept big (none of which are stringent), topological transition would be observable.

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